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A genetic algorithm for fin profile optimization

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Abstract—In the present work a genetic algorithm is proposed in order to optimize the thermal performances of finned surfaces. The bidimensional temperature distribution on the longitudinal section of the fin is calculated by resorting to the finite elements method. The heat flux dissipated by a generic profile fin is compared with the heat flux removed by the rectangular profile fin with the same length and volume. The genetic algorithm is then applied to the case of polynomial profile fins, in order to determine the polynomial parameter values which optimize the fin effectiveness. The optimum profile is finally shown for different polynomial orders.

INTRODUCTION

In many engineering sectors, where high thermal fluxes must be transferred, the finned surface power removers are today an usual tool. Since finned surfaces allow evident improvements in heat transfer effectiveness, the heat exchangers field is one of the most interested in their applications. Moreover new industrial sectors present an increasing interest in the introduction of extended surfaces for heat flux removal. In particular, the electronics industry has promoted a new interest in developing heat removers, aimed at transferring heat from electronic components to the environment, in order to reduce the working temperature and to improve the characteristics and the reliability [1–3].

The optimization of the heat remover longitudinal profile, in order to transfer the highest power with the smallest volume, is a problem that is not yet completely solved. Such a problem was tackled for the first time in the 1920s [4], when Schmidt proposed a parabolic longitudinal profile. Afterwards many authors contested Schmidt's conclusion, correct from the point of view of the utilized model, but scarcely corresponding to the real phenomenon characteristics. Since then many fin profiles have been proposed, mainly parabolic or triangular, but without giving a final solution to the optimization problem [5] and leaving perplexedness regarding the structural integrity of heat removers with an excessively decreasing profile. Recently undulated fin profiles have been proposed [6–8] and a parabolic-undulated fin has been demonstrated as having a noticeably improved effectiveness.

In this work we consider polynomial profile heat removers and we propose a genetic algorithm in order to determine the polynomial parameter values which let the highest power be removed with the same fin volume. The algorithm calculates the heat flux dissipated by the polynomial profile heat remover on the basis of the bidimensional temperature distribution on its longitudinal section, which is obtained with the help of a finite elements model.

For the last few years genetic algorithms have been utilized to solve functions fitting or parameters evaluation problems. Their use in the neural networks training [9] is largely diffused. Just recently they have been utilized in thermosciences [10, 11].

THE FIN MODEL

In the orthogonal coordinate system we will refer to a heat remover with a longitudinal section symmetrical with respect to the x axis and with a profile described by an arbitrary function $f(x)$, as shown in Fig. 1. The fin, with indefinite width and length L, is immersed in a fluid with a constant bulk temperature T_F. Moreover, the fin base temperature T_0 is known.

In order to calculate the heat flux removed by such a fin it is necessary to determine the temperature distribution in the longitudinal section (plane xy). This distribution must satisfy the Laplace's equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial^2 y} = 0 \quad (1)$$

with the boundary conditions:

$$T(0, y) = T_0 \quad (2)$$

$$\left[\frac{\partial T}{\partial x}\right]_{L,y} = -\frac{h_f}{k}[T(L,y) - T_F] \quad (3)$$

NOMENCLATURE

E	compared effectiveness	k	thermal conductivity of the fin [W m^{-1} K^{-1}]
f	half-height of the fin [m]	L	length of the fin [m]
$\bar{f}$	average half-height of the fin [m]	n	polynomial profile order
g_{hi}	thermal conductance between the refrigerant fluid and the elements on the fin surface [W K^{-1}]	Q_d	heat flux removed by the fin examined [W]
g_{ki}	thermal conductance between one element and another [W K^{-1}]	Q_r	heat flux removed by the rectangular fin [W]
g_{0i}	thermal conductance between the elements of the fin base and the ones adjacent [W K^{-1}]	T	temperature of the fin [°C]
		T_0	temperature of the fin base [°C]
		T_F	bulk temperature of the fluid [°C]
h	heat transfer coefficient [W m^{-2} K^{-1}]	x	longitudinal coordinate [m]
h_f	final transversal surface heat transfer coefficient [W m^{-2} K^{-1}]	y	transversal coordinate [m].

$$\left[\frac{\partial T}{\partial y}\right]_{x,0} = 0 \tag{4}$$

$$\left[\frac{\partial T}{\partial y}\right]_{x,f(x)} - \left[\frac{\mathrm{d}f}{\mathrm{d}x}\frac{\partial T}{\partial x}\right]_{x,f(x)} = -\frac{h\sqrt{1+\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_x^2}}{k}[T(x,f(x))-T_F] \tag{5}$$

h and h_f being the convective heat transfer coefficients for the longitudinal fin surface and for the final transversal one, respectively, k being the thermal conductivity of the fin. Due to the complexity of the problem it is convenient to determine the temperature distribution numerically, using for example the finite element method.

We can subdivide the longitudinal section of the fin into an array of elements, by locating some knots that are distant Δx in x direction and Δy in y direction from one another, as shown in Fig. 2. The knots of the outer elements are chosen on the perimeter of the longitudinal section. The elements on the longitudinal outline are such as to follow its shape. The other elements are instead rectangular. Assuming that Δx and Δz are sufficiently short, it is possible to suppose that the temperature varies linearly between two adjacent knots.

Considering the balance between the thermal fluxes which cross the outline of the ith element it is possible to write:

$$\sum_k g_{ki}(T_k - T_i) + g_{0i}(T_0 - T_i) + g_{hi}(T_F - T_i) = 0 \tag{6}$$

T_i and T_k being the temperatures of the ith element and of the elements adjacent to it, g_{ki} being the thermal conductance between the kth and the ith elements, appropriately calculated taking the particular separating surface into account, g_{0i} the thermal conductance between the fin base and the ith element, g_{hi} the thermal conductance between the coolant fluid

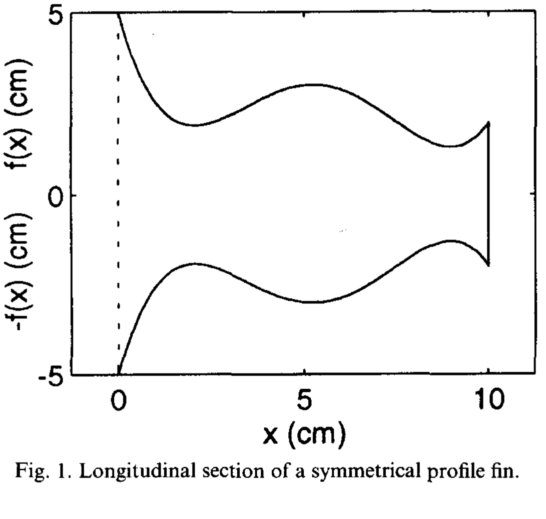

Fig. 1. Longitudinal section of a symmetrical profile fin.

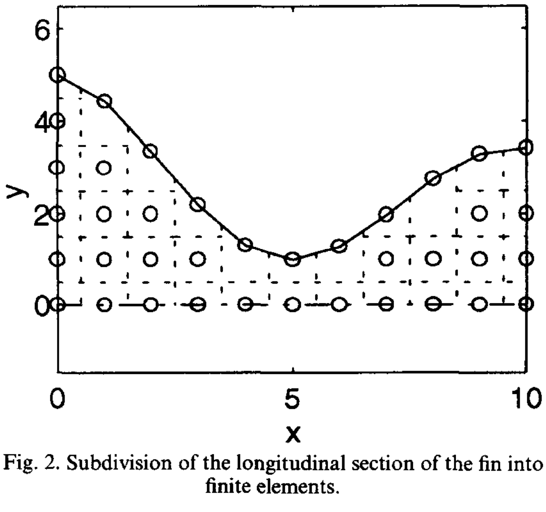

Fig. 2. Subdivision of the longitudinal section of the fin into finite elements.

and the ith element. Conductance g_{0i} is zero for all the elements which are not adjacent to the fin base, while conductance g_{hi} is zero for all the internal elements. Moreover g_{hi} depends on h for the elements on the longitudinal surface, and on h_f for those on the final transversal surface. From equation (6) it follows that:

$$\sum_k g_{ki}T_k - \left(\sum_k g_{ki} + g_{0i} + g_{hi}\right)T_i + g_{0i}T_0 + g_{hi}T_F = 0 \tag{7}$$

that is:

$$\sum_k a_{ik}T_k + a_{ii}T_i = b_i T_0 + c_i T_F. \tag{8}$$

Considering all the elements we can therefore write:

$$\mathbf{A} * \mathbf{T} = \mathbf{B} \times T_0 + \mathbf{C} \times T_F \tag{9}$$

having gathered into vector $\mathbf{T}$ the temperature of all the elements, into matrix $\mathbf{A}$ the elements a_{ik} and into vectors $\mathbf{B}$ and $\mathbf{C}$ the elements b_i and c_i, respectively. It is then possible to determine the temperature distribution on the longitudinal section of the fin by solving the system (9):

$$\mathbf{T} = \mathbf{A}^{-1} * (\mathbf{B} \times T_0 + \mathbf{C} \times T_F). \tag{10}$$

We can now calculate the heat flux dissipated by the remover for unit of width in the following way:

$$Q_d = 2\left(\sum_i g_{0i}(T_0 - T_i) + g_{h0}(T_0 - T_F)\right) \tag{11}$$

g_{h0} being the thermal conductance between the fin base (see Fig. 2) and the coolant fluid.

EFFECTIVENESS OF THE FIN

The fin performances can be evaluated on the basis of the compared effectiveness, i.e. the ratio between the heat flux (Q_d) dissipated by the heat remover with a generic profile and the heat flux (Q_r) removed by a fin of the same volume and length and with rectangular profile:

$$E = \frac{Q_d}{Q_r}. \tag{12}$$

Let us then consider a rectangular fin of width $2\bar{f}$, $\bar{f}$ being the average value of $f(x)$:

$$\bar{f} = \frac{1}{L}\int_0^L f(x)\,\mathrm{d}x. \tag{13}$$

The temperature distribution on the longitudinal section of such a fin must satisfy equation (1), the boundary conditions (2)–(4) and the following:

$$\left[\frac{\partial T}{\partial y}\right]_{x,f(x)} = -\frac{h}{k}[T(x,f(x)) - T_F]. \tag{14}$$

Since both longitudinal and final transversal surfaces are plane we can assume h equal to h_f. By integrating equation (1) with the above boundary conditions the following solution is obtained [12]:

$$T(x,y) = T_F + \frac{2h(T_0 - T_F)}{k}\sum_{n=1}^{\infty}\left[\frac{1}{\left(\alpha_n^2 + \frac{h^2}{k^2}\right)\bar{f} + \frac{h}{k}}\frac{\cos(\alpha_n y)}{\cos(\alpha_n \bar{f})} \times \frac{\alpha_n \cosh[\alpha_n(L-x)] + \frac{h}{k}\sinh[\alpha_n(L-x)]}{\alpha_n \cosh(\alpha_n L) + \frac{h}{k}\sinh(\alpha_n L)}\right] \tag{15}$$

being α_n the solutions of the equation:

$$\alpha \tan(\alpha \bar{f}) = \frac{h}{k}. \tag{16}$$

The heat flux dissipated for unit of length is then:

$$Q_r = 4h(T_0 - T_F)\sum_{n=1}^{\infty} \times \left[\frac{\tan(\alpha_n \bar{f})}{\left(\alpha_n^2 + \frac{h^2}{k^2}\right)\bar{f} + \frac{h}{k}} \frac{\alpha_n \sinh(\alpha_n L) + \frac{h}{k}\cosh(\alpha_n L)}{\alpha_n \cosh(\alpha_n L) + \frac{h}{k}\sinh(\alpha_n L)}\right]. \tag{17}$$

THE GENETIC ALGORITHM

We now propose a genetic algorithm which is able to determine the values of the fin profile describing parameters which allow the highest compared effectiveness. We will consider heat removers for which the profile function $f(x)$ has a polynomial form:

$$f(x) = \sum_{j=0}^{n} a_j x^j. \tag{18}$$

By increasing the polynomial order n, more and more articulate fin profiles will be taken into account.

Genetic algorithms are usually utilized in order to assign the values which allow a particular performance to the parameters of a system. A genetic algorithm begins with a population of several samples of the same system with parameters assigned arbitrarily or randomly. The different samples undergo a set of trials in order to compare their performances with the required one. In this way each sample receives an evaluation. Samples which have obtained the best evaluations are then selected and reproduced with random mutations in order to compose a new generation. The random mutations reproducing process consists of copying the parameter values adding little random changes. The new generation undergoes the same set of trials, it is selected and reproduced. The process continues until performances which are near enough to the required one are found or until the samples evaluations stop improving.

In the proposed algorithm the system is the fin, and the parameters are the quantities which describe its profile. The required performance consists of having the highest compared effectiveness possible.

After choosing as a constant the order of the polynomial function which describes the fin profile, a rectangular profile fin with an appropriate height is chosen as a prototype. The prototype is then reproduced with random mutations uniformly distributed between -10% and $+10\%$, in order to compose an initial population of 100 samples (including the prototype). As an evaluation the compared effectiveness is assigned to each sample. The 20 samples with the best evaluations are selected and reproduced with the mutation rule described above. The new generation is evaluated, selected and reproduced in the same way. The process continues until there is no more improvement in the compared effectiveness of the best sample.

The population dimension would have to be chosen on the basis of the polynomial order. With little orders very numerous populations are not required to keep the algorithm from stopping in correspondence with a local maximum. The proposed 100 samples dimension allows the algorithm to be easily utilized in every case we considered.

As fin profile describing parameters not the polynomial coefficients, but the values of the polynomial function in $n+1$ equidistant points, placed between the fin base and final section, are chosen. In this way the parameters have the same dimensions, and their changes influence the compared effectiveness with more comparable weight. Such a characteristic makes the parameter modification by the genetic algorithm easier and reduces the possibility of the algorithm stopping in correspondence with only a local maximum in the compared effectiveness.

In the algorithm it is possible to impose an upper and a lower limit, y_{max} and y_{min}, to the fin profile by rescaling the parameters before evaluating the performances in the following way. Let p_k be the parameters of a polynomial profile which is found not to respect a limit and let f_{max} and f_{min} be maximum and minimum value of the profile. The rescaling is then:

$$p'_k = f_{min} + \frac{y_{max} - f_{min}}{f_{max} - f_{min}} (p_k - f_{min})$$

$$k = 1,2 \ldots n+1 \quad \text{if } f_{max} > y_{max} \qquad (19)$$

$$p'_k = f_{max} + \frac{f_{max} - y_{min}}{f_{max} - f_{min}} (p_k - f_{max})$$

$$k = 1,2 \ldots n+1 \quad \text{if } f_{min} < y_{min} \qquad (20)$$

p'_k being the new parameter values.

Another way to let the optimum profile respect an upper and a lower limit consist in assigning a null evaluation in the algorithm to the samples whose profile does not satisfy the requirements. But in this manner a good profile which exceeds the limits for a very short distance can be lost. For this reason, in the genetic optimization examples that were shown in the present article we followed the first method, which is, in fact faster.

In order to make the genetic algorithm even faster, it is possible to build at the beginning matrices **A**, **B** and **C**, regarding a fin with $f(x)$ constant and equal to y_{max}. Afterwards to evaluate the performance of each sample these matrices are copied and only a few of their elements are modified. In particular, it is necessary to change the value of the conductances for the finite elements placed on the fin outline and for those adjacent.

After establishing the physical parameters of the system it is useful also to tabulate the values of Q_r as a function of $\bar{f}$. In this manner in the genetic algorithm only the time necessary to calculate $\bar{f}$ for each sample is required.

RESULTS

The proposed genetic algorithm has been utilized in order to optimize the polynomial profile of aluminium fins. For the finite element model parameters the values reported in Table 1 have been assumed.

Coefficient h has first been assumed constant and equal to 30 W m^{-2} K^{-1}, which is an average value obtainable for wavy surfaces with air in forced convection [13]. The same value has also first been assigned to h_f.

The algorithm was utilized by choosing the order of the polynomial function which describes the fin profile equal to a value from 1 to 5. To the prototype a 2.5 cm half-height was assigned. We imposed the half-height of the reproduced fin samples to be neither more than 5 cm nor less than 0.5 cm. An upper limit was required since the algorithm tended to increase the maximum height of the fin. The compare effectiveness, in fact, always grows with the fin height. The lower limit was instead assigned in order to assure the physical consistence of the fin.

The polynomial profiles obtained with the genetic algorithm are reported in Fig. 3 together with the relative compared effectiveness. After the third polynomial order an undulate pattern is evident. The highest and the lowest values of the fin profile fit on the assigned limit. Since coefficient h is not very high, the algorithm tends, in fact, to extend the longitudinal thermal exchange surface as much as possible. At the final fin section in each case the profile assumes the highest value. That occurs because the heat is sup-

Table 1. Values assigned to the parameters of the finite element model.

$L = 0.1$ m	$k = 200$ W m^{-1} K^{-1}
$T_0 = 70°C$	$T_F = 20°C$

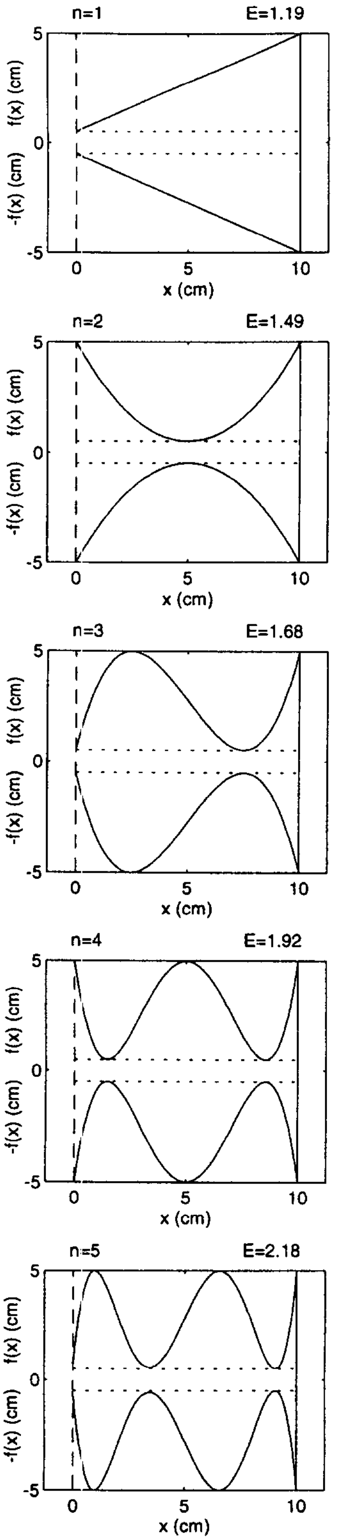

Fig. 3. Polynomial profiles which optimize the compared effectiveness of an aluminium fin when h is equal to 30 W m^{-2} K^{-1}. Dotted horizontal lines represent the lower limit for the fin height.

posed to be removed from the final transverse surface too.

Afterwards we utilized the genetic algorithm by assuming that h and h_f were both equal to 100 W m^{-2} K^{-1}. The polynomial profiles obtained in such a situation are shown in Fig. 4. Since the convective heat transfer coefficient is higher than in the previous case, the requirement of extending the transfer surface as much as possible is now less important, compared to that of making the longitudinal thermal conduction easier. For this reason, while the highest values of the fin profile still fit on the assigned limit, the local minima, mainly in the region close to the fin base, are higher than in the previous case.

In order to better understand the compromise between the requirement of extending the heat transfer surface as much as possible and that of making the longitudinal thermal conduction easier, the temperature distribution on the longitudinal section of the fin with an optimum fourth order polynomial profile is shown in Figs. 5 and 6 with h equal to 30 W m^{-2} K^{-1} and to 100 W m^{-2} K^{-1}, respectively. In the second case the temperature drop between the base and the final section of the fin is higher, because of the better cooling conditions. Moreover, in the first case strong reduction in the fin hight causes smaller longitudinal changes in temperature.

In Fig. 7 the highest values obtained for the compared effectiveness are shown vs the order of the polynomial profile both with h equal to 30 W m^{-2} K^{-1} and to 100 W m^{-2} K^{-1}. In both cases an increasing trend is evident. Obviously, the compared effectiveness cannot decrease with the polynomial order, since lower order polynomial functions are approximated by higher order ones. For this reason, on the other hand, in the genetic algorithm we did not include an optimization of the polynomial profile order either. In the considered domain the compared effectiveness increases in a nearly linear manner with the polynomial order and no imminent saturation is preannounced. Moreover, for each polynomial order the compared effectiveness is higher when the convective heat transfer coefficient is lower. In this situation, in fact, the benefits of extending the transfer surface are more significant than the disadvantages of reducing the longitudinal conduction. A wavy profile, compared with a rectangular one, is then less convenient when the convection coefficient is higher.

The polynomial parameters of the obtained profiles are finally reported in Tables 2 and 3. It must be remembered that these parameters represent the fin half-height in $n+1$ equidistant point, placed between the base and the final section of the fin.

CONCLUSIONS

The genetic algorithm proposed seems able to resolve the problem of optimizing the longitudinal profile of a fin, in order to improve its performances

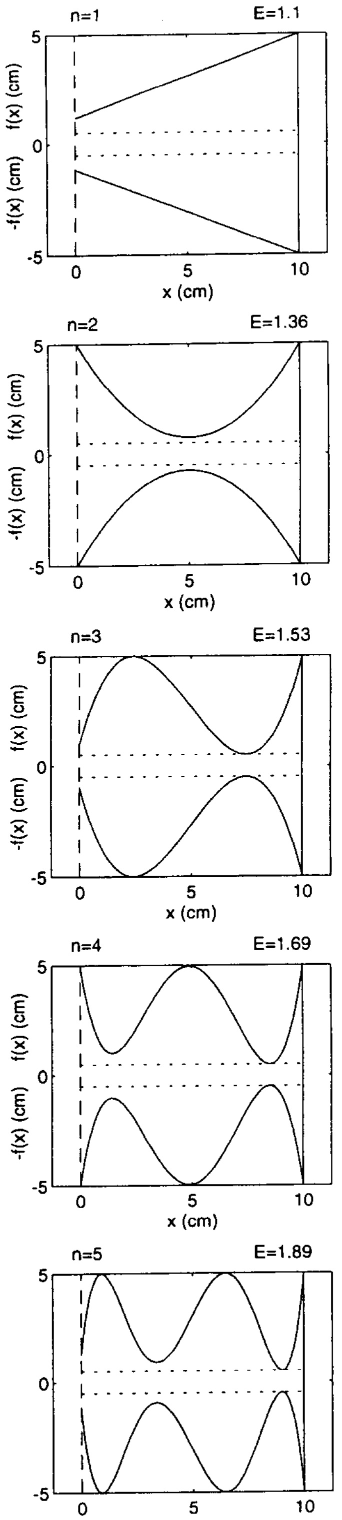

Fig. 4. Polynomial profiles which optimize the compared effectiveness of an aluminium fin when h is equal to 100 W m^{-2} K^{-1}. Dotted horizontal lines represent the lower limit for the fin height.

compared with those of a rectangular longitudinal section fin of the same volume and length.

The optimization examples shown in the article demonstrate that it is possible to noticeably increase the compared effectiveness of a fin by introducing undulations in its profile when the convective heat transfer coefficient is not very high. In such a condition, for example, a fin with a fourth order polynomial profile, even though it is not yet too difficult to build for the producer, can remove nearly twice as much heat flux as that dissipated by a rectangular fin of the same volume. On the other hand, under better convection conditions the convenience of an undulate profile fin decreases.

A more correct solution for the problem of optimizing the heat removers profile will be obtained with the genetic algorithm proposed by considering a local convective heat transfer coefficient which is variable in length (or in width for the final transverse section). In particular, it will be interesting to take the changes in the local convection coefficient induced by the variation of the profile into account. In the case of the optimum fifth order polynomial profile, for example, a very narrow channel is created at the base of the fin. In this region the convection coefficient, in the practice, can therefore assume a value which is noticeably lower than the average along the fin surface. The amount of surface gained in this region is then less worth. Since in general the changes in the local convection coefficient along the fin surface increase with the profile oscillations, in the genetic optimization examples presented we did not consider polynomial orders greater than the fifth, which describe more undulate profiles. Nevertheless, after having calculated the local convection coefficient for a generic profile fin, it is possible to also take this parameter into account to optimize the performances of heat removers by using the same finite element model and genetic algorithm proposed.

Finally, it must be noticed that in the genetic fin profile optimization examples presented a constant temperature has been assigned to the base of the fin for each value of y coordinate. In practical applications the base temperature of the fin cannot always be assumed to be a constant. In many problems, in fact, the entity assigned is the heat flux, which is to be removed from a wall surface with the help of fins. In such a condition, if the fins are thin and largely spaced, noticeable temperature variations occur on the surface to be cooled and, in particular, at the fin base. Neglecting this variation can result in errors of more than 20% in calculating the heat flux removed by the fin [14, 15]. In order to obtain a more correct solution for the problem of optimizing the profile of the fins under the above quoted conditions, it is then convenient to utilize the genetic algorithm proposed, employing a finite element model which reproduces also a portion of the wall with heat flux assigned to the side opposite to the fins.

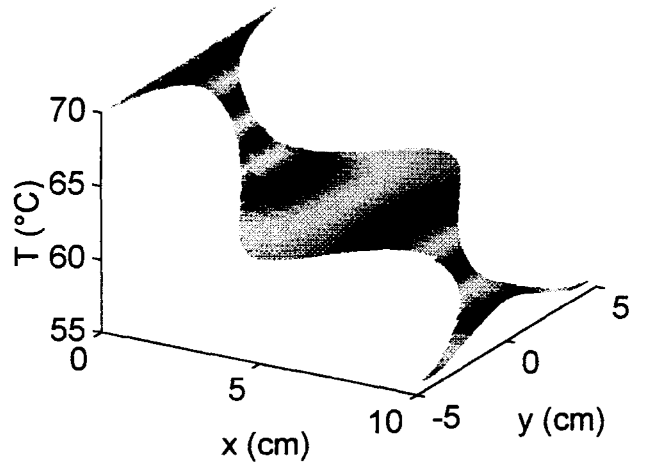

Fig. 5. Temperature distribution on the longitudinal section obtained for an optimized fourth order polynomial profile fin with h equal to 30 W m^{-2} K^{-1}.

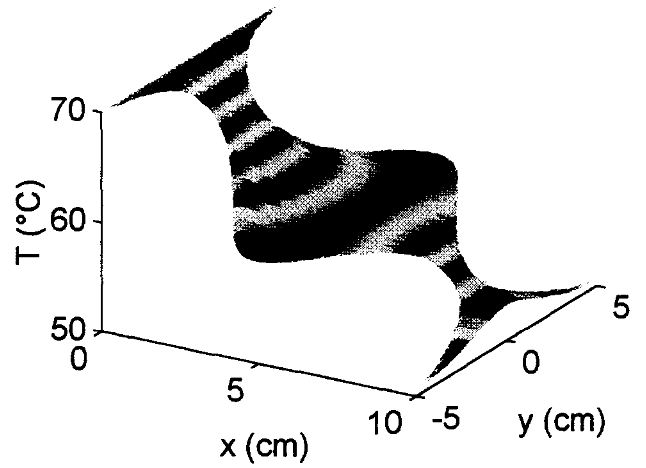

Fig. 6. Temperature distribution on the longitudinal section obtained for an optimized fourth order polynomial profile fin with h equal to 100 W m^{-2} K^{-1}.

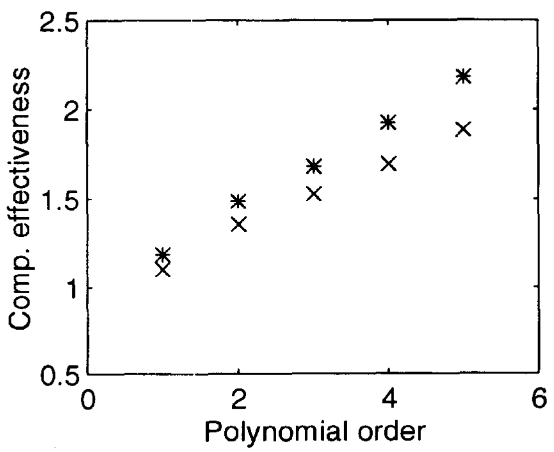

Fig. 7. Highest values of the compared effectiveness obtained for different polynomial orders with h equal to 30 W m^{-2} K^{-1} (stars) and 100 W m^{-2} K^{-1} (cross).

Table 2. Profile describing parameters which optimize the compared effectiveness when h is equal to 30 W m^{-2} K^{-1}

	Profile describing parameters [cm]					
$n = 1$	0.51					5
$n = 2$	5		0.5			4.99
$n = 3$	0.54	4.66		0.84		5
$n = 4$	4.99	1.62	4.99		1.62	4.99
$n = 5$	0.7	2.92	0.93	4.67	2.55	5

Table 3. Profile describing parameters which optimize the compared effectiveness when h is equal to 100 W m^{-2} K^{-1}

	Profile describing parameters [cm]					
$n = 1$	1.16					5
$n = 2$	5		0.74			5
$n = 3$	0.96	4.6		0.81		4.98
$n = 4$	5	2.09	4.95		1.53	4.99
$n = 5$	1.3	3	1.31	4.76	2.46	4.98

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